Statistics

Lecture 13

Feb 19-8:47 AM

Binomial Prob. Dist.: (SG 16)

Consider a binomial Prob. dist with $n=80$ and $P=.3$.

1) $q = 1-P = .7$

2) $np = 80(.3) = 24$

3) $npq = 80(.3)(.7) = 16.8$

4) $\sqrt{npq} = \sqrt{16.8} \approx 4$

5) P(exactly 25 Successes)

$P(x=25) = \text{binompdf}(80, .3, 25) = .093$

6) P(at most 30 Successes)

$P(x \le 30) = \text{binomcdf}(80, .3, 30) = .941$

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7) P(at least 20 successes)

$P(x \geq 20) = 1 - P(x \leq 19)$

we don't want: 19 and below; we want: 20 and above

$= 1 - \text{binomcdf}(80, .3, 19) = .865$

8) P(between 18 and 30 successes, inclusive)

$P(18 \leq x \leq 30) = P(x \leq 30) - P(x \leq 17)$

$P(x \leq 30)$: $P(x \leq 17)$ | 17 | 18 | 30

$= \text{binomcdf}(80, .3, 30) - \text{binomcdf}(80, .3, 17)$

$= .888$

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Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard Deviation $\sigma = \sqrt{\sigma^2}$

Binomial Prob. Dist.

Suppose you guess on 100 True/False questions. Success is to guess correctly.

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $\mu = np = 50$ 5) $\sigma^2 = npq = 25$ 6) $\sigma = \sqrt{\sigma^2} = 5$

7) 68% Range $\mu \pm \sigma = 50 \pm 5 \Rightarrow$ 45 to 55

8) 95% Range (usual Range) $\mu \pm 2\sigma = 50 \pm 2(5) \Rightarrow$ 40 to 60

9) P(guess at most 60 correct answers)

$P(x \leq 60) = \text{binomcdf}(100, .5, 60) = .982$

10) P(guess at least 45 correct answers)

$P(x \geq 45) = 1 - P(x \leq 44)$

we don't want: 44 and below; we want: 45 and above

$= 1 - \text{binomcdf}(100, .5, 44) = .864$

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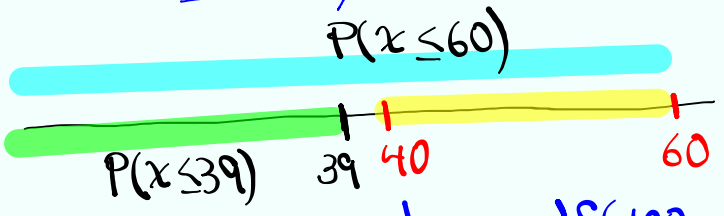
11) P(guess correctly between 40 & 60 inclusive Correct answers.)

$P(40 \le x \le 60) = P(x \le 60) - P(x \le 39)$



= binomcdf(100, .5, 60) − binomcdf(100, .5, 39)

≈ .965 96.5%

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Suppose You guess on a multiple-choice exam with 400 questions and each question has 5 choices with only one Correct choice.

1) $n = 400$

2) $p = \frac{1}{5} = .2$

3) $q = \frac{4}{5} = .8$

4) $\mu = np = 80$

5) $\sigma^2 = npq = 64$

6) $\sigma = \sqrt{\sigma^2} = 8$

7) Usual Range (95%) $\mu \pm 2\sigma \Rightarrow$ 64 to 96

8) P(guess exactly 100 correct answers)

$P(x = 100) = \text{binompdf}(400, .2, 100)$

$= .002$

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9) P(guess Correctly on fewer than 100 Correct answers)

$P(x < 100) = P(x \leq 99) = \text{binomcdf}(400, .2, 99)$

99 100

$= .991$

10) P(guess more than 70 Correct Ans.)

$P(x > 70) = P(x \geq 71) = 1 - P(x \leq 70)$

we don't want $P(x \leq 70)$ 70 71 we want

$= 1 - \text{binomcdf}(400, .2, 70)$

$= .884$

11) P(guess Correctly between 64 & 96, inclusive)

$P(64 \leq x \leq 96) = P(x \leq 96) - P(x \leq 63)$

64 96

$= \text{binomcdf}(400, .2, 96) - \text{binomcdf}(400, .2, 63)$

$= .961 \approx 96\%$

SG 16

on page 4

Make Sure to use P & q in fraction.

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Prob. dist. with Continuous random Variable:

SG 17-20

1) Uniform Prob. dist.

2) Standard Normal Prob. dist.

3) Normal Prob. dist.

4) Central Limit Theorem

5) Applications

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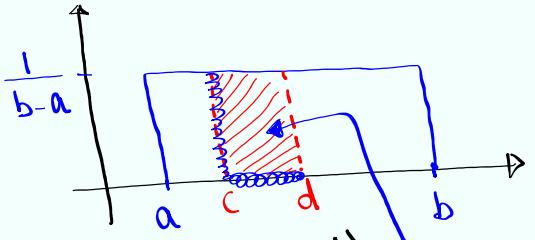
Uniform Prob. dist.:

Let x be a continuous random variable for all values from a to b with uniform prob. dist.

Graph is rectangular as shown below with total area 1.



$P(x=c) = 0$ Line has 0 area.

$P(c < x < d) = (d-c) \cdot \frac{1}{b-a}$

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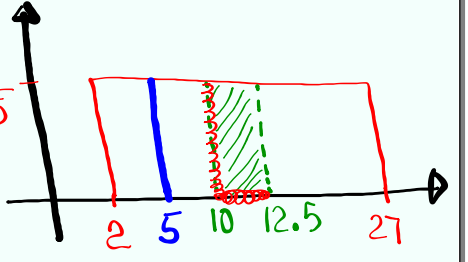
Consider a uniform Prob. dist. for all values from 2 to 27.

$\frac{1}{27-2} = \frac{1}{25}$



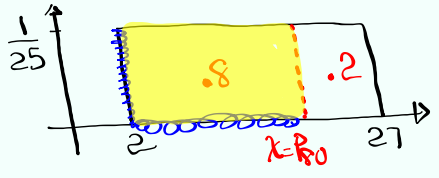
$P(x=5) = 0$

$P(10 < x < 12.5) = (12.5-10) \cdot \frac{1}{25} = \frac{2.5}{25} = \frac{1}{10}$

Find $x = P_{80}$

80% below

20% above



$(x-2) \cdot \frac{1}{25} = .8$

$x - 2 = 25(.8)$

$x = 2 + 25(.8) = 22$

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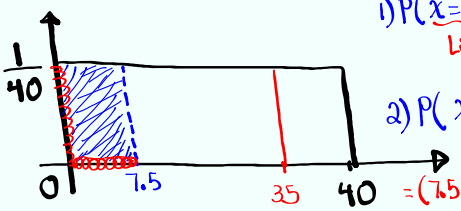
Consider a uniform Prob. dist. for all values from 0 to 40.



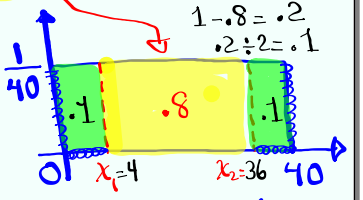
1) $P(x=35)=0$ Line

2) $P(x<7.5)$

$=(7.5-0)\cdot\frac{1}{40}=\frac{7.5}{40}$

$=\boxed{\frac{3}{16}}$

3) Find two values that separate the middle 80% from the rest.



$1-.8=.2$

$.2\div 2=.1$

$(x_1-0)\cdot\frac{1}{40}=.1$

$x_1-0=40(.1)$

$\boxed{x_1=4}$

$(40-x_2)\cdot\frac{1}{40}=.1$

$40-x_2=40(.1)$

$40-x_2=4$

$40-4=x_2$

$\boxed{x_2=36}$

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Standard Normal Prob. Dist.:

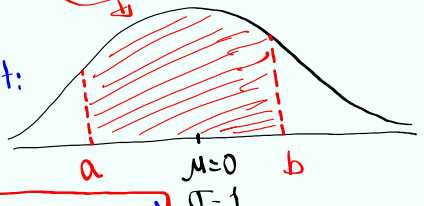
1) use Z, $P(Z=c)=0$

2) Dist. is symmetric, bell-shape with total area 1.

3) Mean = Mode = Median

4) $\mu=0$, $\sigma=1$

$P(a<Z<b)$ is the corresponding area within the graph as shown below.



How To Find it:

2nd VARS

normalcdf(L, U, μ, σ)

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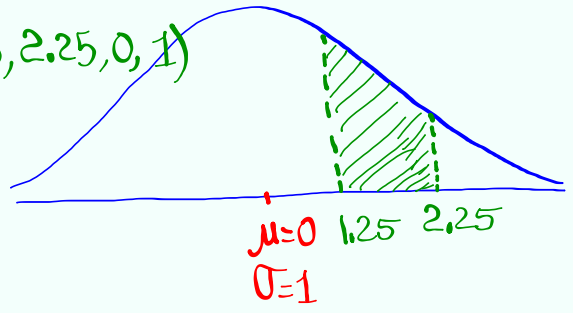
$P(1.25 < Z < 2.25)$

= normalcdf(1.25, 2.25, 0, 1)

= .093

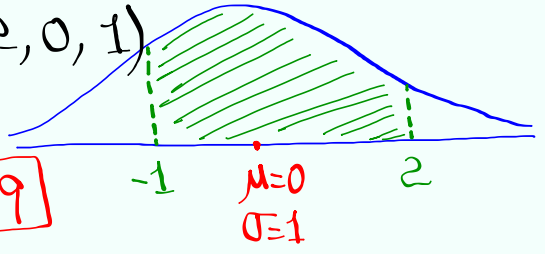


$P(-1 < Z < 2)$

= normalcdf(−1, 2, 0, 1)

(−)

= .819



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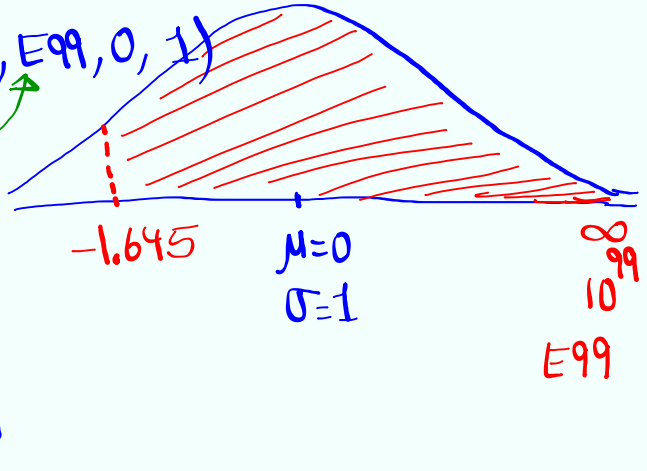
$P(Z > -1.645)$

= normalcdf(−1.645, E99, 0, 1)

(−)

2nd 9

= .950



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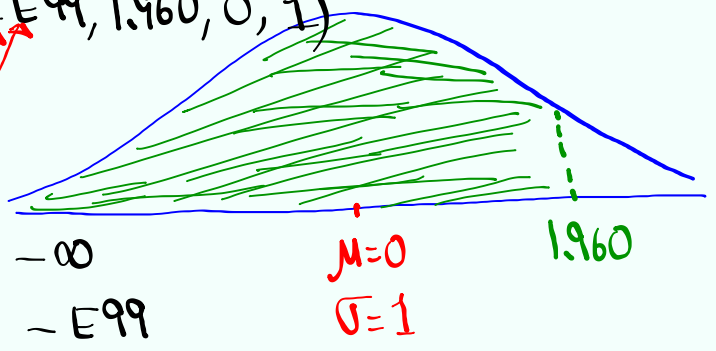
$P(Z < 1.960)$

$= \text{normalcdf}(-E99, 1.960, 0, 1)$

(-) 2nd [2]



$= .975$

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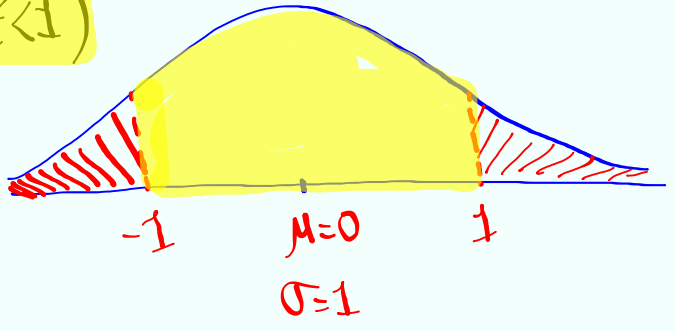
$P(Z < -1 \text{ or } Z > 1)$ is it Wan and $= 0$

$= 1 - P(-1 < Z < 1)$

Total Area
Total Prob.



$= 1 - \text{normalcdf}(-1, 1, 0, 1) = .317$

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Doing Reverse:

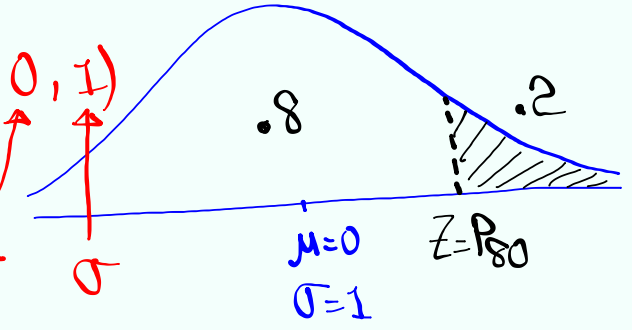
Find $z = P_{80}$

80% below → .8 Left Area

20% above → .2 Right Area

$z = \text{invNorm}(.8, 0, 1)$

left Area, μ, σ



$\approx$ .842

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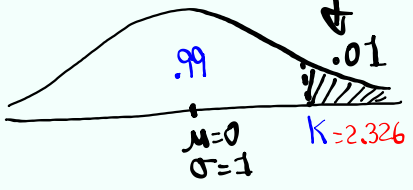
Find K such that $P(z > K) = .01$

Right Area

$K = \text{invNorm}(.99, 0, 1)$

≈ 2.326

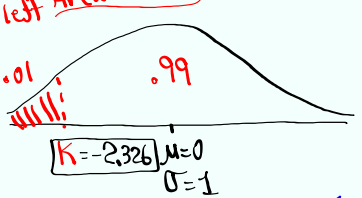


Find K such that $P(z < K) = .01$

left Area

$K = \text{invNorm}(.01, 0, 1)$

$= -2.326$



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